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Investigation of Nonstandard Finite Difference for Fractional Order Covid-19 Model

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Highlights

- This article focuses on the mathematical model of the Covid 19 epidemic disease.
- In the study, NSFD scheme approach was proposed to solve the Covid 19 model.
- The effects of the fractional derivative were correctly obtained in the numerical solutions.

Article Info

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Keywords

Caputo fractional derivative, Covid 19 model, Fractional order nonstandard finite difference, Grünwald-Letnikov

Abstract

This article examines a mathematical model of the Covid-19 type. We demonstrate how the population is impacted by immigration, protection, the mortality, exposure, curing, and interactions between sick and healthy individuals. There are five classifications in our model: exposed, susceptible, infected, quarantined, and recovered. The model is subjected to numerical and fractional analysis in this instance. The numerical analysis is performed using the fractional order non-standard finite difference (NSFD) scheme. The Grunwald-Letnikov numerical approximation technique is used for fractional analysis. The findings are evaluated by simulations using the Matlab tool.

1. INTRODUCTION

Epidemiology is known as the science of epidemic diseases, which is one of the basic disciplines dealing with public health. To define it more broadly, epidemiology (epidemic) is a field of study for the purpose of investigating the responsible factors that cause diseases to occur and even spread, in a certain time and environment. Today's mathematical epidemiology has become a very useful science in estimating the recurrence and spread of epidemics, incidence or mortality rates, and in investigating the cause of disease.

An important part of today's mathematical modeling studies are models related to epidemic diseases. Many centers such as the World Health Organization, 'Centre for the Mathematical Modeling of Infections Diseases' (England) and 'BC Center for Disease Control' (Canada) carry out intensive studies on the modeling of diseases.

Many fields, including biology, chemistry, psychology, and economics, use mathematical modeling of real-world occurrences. Financial models, for example, frequently include stochastic equations, whereas ordinary differential equation systems are the fundamental instruments for predicting the spread of infectious diseases. The use of fractional analysis should be especially highlighted because the current

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tendency in mathematical modeling is to use various forms of fractional derivatives with several solution methods to find the precise, approximate, or numerical solutions of the system under consideration. Given the limited knowledge and underreporting of the Covid-19 illness, it is challenging to accurately evaluate characteristics such as the rate of transmission, recovery, and contact rates. Therefore, the use of classical differential equations for modeling may not be appropriate for accurately depicting the dynamics of the populations in question. Furthermore, the use of fractional equations can be justified by the ability to modify the order of the differential equation based on the real facts on the propagation of the disease.

Compartment models for epidemiology are models that began to be studied in the early 1900s and examine the dynamics of transmission [1-3]. These models have been made quite reasonable estimates for many [4-6] and today these models are also used for the Covid-19 [7, 8]. Compartment models divide a closed society into segments called compartments. Each compartment is denoted by a capital letter: for example, S represents the total number of people at risk, E the total number of people infected but not yet contagious, I the total number of people infected (and contagious), and R the total number of recovered people. The model gets its name from the compartments, and the compartments are named after the individual's disease-related states.

By using the SIR model, which is called the classical compartmental model, new models can be created to model different diseases and conditions. For example, if the patient has been exposed to the disease but has not yet become active in the body, the SEIR model is formed by adding an edible compartment E (exposed). Similarly, by adding different situations, epidemic disease models consisting of different compartments such as SIS, SEIR, SEIRS, MSEIR can be derived.

Fractional calculus offers a robust mathematical framework for the modeling of systems and processes that exhibit memory effects and nonlocal behavior, which are frequently not represented by conventional integer-order differential equations. This method facilitates the modeling of systems with history-dependent dynamics by incorporating fractional derivatives and integrals, which are generalizations of classical calculus. Non-integer DE structures provide the advantage of integrating the memory effect into the model and allowing for higher degrees of freedom. We use the fractional Grünwald-Letnikov (GL) technique [9-11] with Binomial coefficients on the fractional Covid 19 mathematical model. The significance of GL operators extends beyond historical and theoretical considerations, encompassing the potential for direct implementation in numerical computations, via their provision of a discretization scheme. Fractional calculus offers a robust mathematical framework for the modeling of systems and processes that exhibit memory effects and nonlocal behavior, which are frequently not represented by conventional integer-order differential equations. This method facilitates the modeling of systems with history-dependent dynamics by incorporating fractional derivatives and integrals, which are generalizations of classical calculus.

The objective of this study is to use the fractional-order non-standard finite difference (NSFD) technique to quantitatively solve Covid-19 models in the time domain. We design an efficient non-standard finite difference (NSFD) scheme to implement the model and study its complex behavior. The primary objective of this study is to create a numerical scheme that will generate approximate solutions for the fractional COVID-19 model. Here, we provide a method that utilizes fractional differences, which may be seen as an inherent extension of Grünwald-Letnikov (GL) operators, a commonly employed family of operators in fractional computing. This work employs a mathematical model that seeks to flatten the infection curve, drawing inspiration from the continuous community transmission of the new COVID-19 viral infection. In order to accomplish this objective, we employ a novel five-compartment fractional mathematical model that is grounded on the epidemiological condition of people.

Recently, a lot of work has been done on Fractional Epidemiological models [12, 13]. Caputo and Riemann-Liouville fractional derivatives [14-16] are the most important definitions of the fractional derivative. Recently, the relationship between modeling and literature on fractional derivatives, COVID-19 and the aforementioned real-life problems and the comparison results are given in the relevant recommended articles [17-28]. Stability analysis of mathematical models is widely recognized as crucial in the fields of applied mathematics. Due to the discrete nature of points in real-life issues, it is crucial to discretize the models in stability analysis. Likewise, employing conventional numerical techniques to

address such issues might result in numerical instability. Hence, unconventional approaches are crucial. Among the discrete approaches, the Non-Standard Finite Difference (NSFD) method, which was created by Mickens [29-33], is highly efficient and straightforward to apply. Furthermore, it gives evidence of convergence even when using greater step sizes. A comprehensive literature review of NSFD schemes may be found in the publications of Patidar [33-35]. NSFD systems have been extensively studied in several fields of practical mathematics. NSFD schemes and stability analysis have been the subject of much recent study [35-41].

This paper presents a numerical analysis of the Covid-19 model using the non-standard finite difference method. An investigation of fractional analysis behaviors is conducted using the Grunwald-Letnikov method. Specific definitions can be found in section 2. The third section comprises the non-standard finite difference method and the corresponding applied process. The fourth section is dedicated to the application of the approach, including both graphical and numerical answers.

2. MATERIAL METHOD

Definition 1. [14-15]

The Caputo fractional derivative of order $\beta > 0$ of a function $f: (0, \infty) \to \mathbb{R}$ is given by

$$ {}_0^C D_t^\beta [\, f(t)] = \begin{cases} \frac{1}{\Gamma(n-\beta)} \int_0^t (t-\eta)^{n-\beta-1} f(\eta)^{(n)} d\eta, 0 \le n-1 < \alpha < n, n = [\beta], n \in \mathbb{N} \\ \left(\frac{d}{dt}\right)^n f(t), \beta = n, n \in \mathbb{N} \end{cases} . $$

Definition 2. The discretization of Fractional derivative by the Grünwald-Letnikov approach

$$ {}_0^C D_t^\beta [\, f(t)] \,|_{t=t^\zeta} = \frac{1}{t^\alpha} \left(f_{\zeta+1} - \sum_{i=1}^{\zeta+1} u_i f_{\zeta+1-i} - v_{\zeta+1} f_0 \right) $$

where $\zeta = 1,2, \dots, N_\zeta$ and the coordinate of each mesh point is $t^\zeta = \zeta \Delta t, \Delta t = \frac{T_k}{N_\zeta}$, $u_i = (-1)^{i-1} \binom{\beta}{i}, u_1 = \beta, v_i = \frac{i^\beta}{\Gamma(1-\beta)}$ and $i = 1,2,3, \dots, \zeta + 1$.

Also let us presume that $0 < u_{i+1} < u_i < \cdots < u_1 = \beta < 1, 0 < v_{i+1} < v_i < \cdots < v_1 = \frac{1}{\Gamma(1-\beta)}$.

Definition 3. The Riemann-Liouville fractional derivative of order $0 < \beta < 1$ of a function is defined as [14]

$$ {}_0^{RL} D_x^\beta f(x) = \frac{1}{\Gamma(\beta-1)} \frac{d}{dx} \int_0^x (x-t)^{-\beta} f(t) dt \, . $$

where $\Gamma(.)$ denotes the Gamma function.

2.1. Formulation of Covid 19 Mathematical Model

This section presents a mathematical method for the population during COVID-19. The population is divided into five divisions [35]. S is susceptible, E is exposed, I becomes infected, Q is quarantined, and R recovers. The system's dynamics are represented by the differential equations shown below.

$$ \frac{dS}{dt} = \Lambda - \phi_c IS - \mu S + v_3 R, $$

$$\begin{aligned}
\frac{dE}{dt} &= \phi_c IS - (\sigma + \mu)E, \\
\frac{dI}{dt} &= \sigma \mathrm{E} - (\alpha + v_2 + \delta + \mu)\mathrm{I} + v_1 R, \\
\frac{dQ}{dt} &= \alpha I - (\theta + \mu)Q, \\
\frac{dR}{dt} &= \theta Q - (v_1 + v_3 + \mu)R + v_2 I.
\end{aligned} \tag{1}$$

The dynamics of the model (1) above are depicted in the chart below.

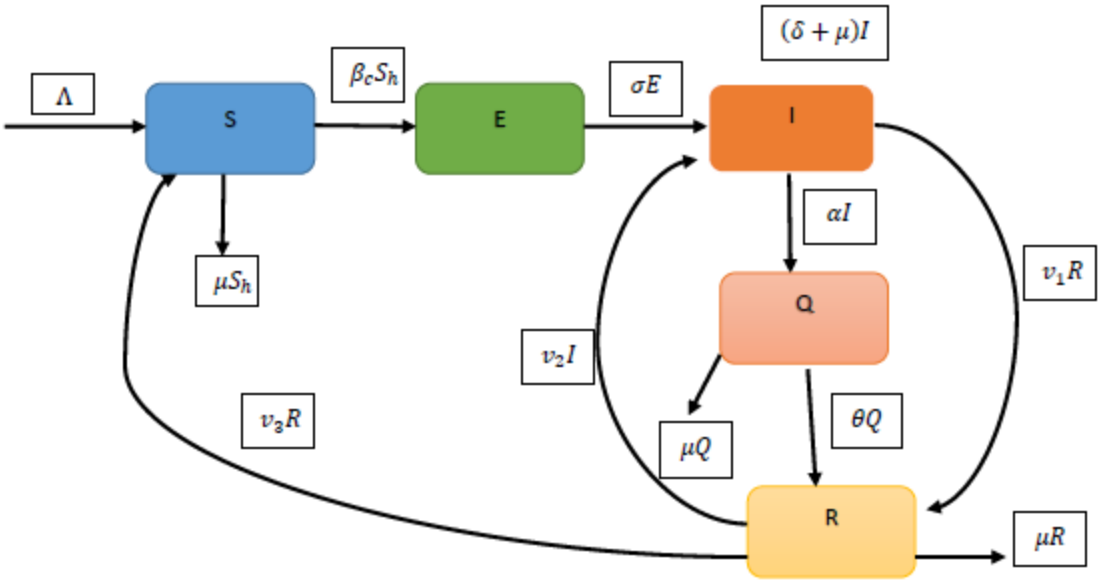


Figure 1. *Flow Chart of Covid-19 Model [42]*

Table 1. *Variables in the model [42] shown in (1)*

Variable	Description
$\boldsymbol{S(t)}$	Total number of vulnerable people at time t
$\boldsymbol{E(t)}$	Number of exposed people at time t
$\boldsymbol{I(t)}$	The number of infected individuals at time t
$\boldsymbol{Q(t)}$	Number of persons in quarantine at time t
$\boldsymbol{R(t)}$	Humans that have been recovered as of time t

Figure 1 shows the Flow chart of the Covid 19 model and Table 1 introduces the variables in the model. The following nonlinear system can be used to represent the Covid-19 fractional order partition epidemic model analytically:

$$\begin{aligned}
{}_0D_t^{\beta} S(t) &= \Lambda^{\beta} - \phi_c^{\beta} I(t)S(t) - \mu^{\beta} S(t) + v_3^{\beta} R(t), \\
{}_0D_t^{\beta} E(t) &= \phi_c^{\beta} I(t)S(t) - \left(\sigma^{\beta} + \mu^{\beta}\right)E(t), \\
{}_0D_t^{\beta} I(t) &= \sigma^{\beta}\mathrm{E}(t) - \left(\alpha^{\beta} + v_2^{\beta} + \delta^{\beta} + \mu^{\beta}\right)\mathrm{I}(t) + v_1^{\beta} R(t), \\
{}_0D_t^{\beta} Q(t) &= \alpha^{\beta} I(t) - \left(\theta^{\beta} + \mu^{\beta}\right)Q(t), \\
{}_0D_t^{\beta} R(t) &= \theta^{\beta} Q(t) - \left(v_1^{\beta} + v_3^{\beta} + \mu^{\beta}\right)R(t) + v_2^{\beta} I(t).
\end{aligned} \tag{2}$$

Table 2. *Parameter values used in the model [42, 43]*

Parameter	Definition	Value
$\boldsymbol{\Lambda^{\beta}}$	Rate of recruitment of individuals	750
$\boldsymbol{\phi_c^{\beta}}$	Rate of Contact for COVID-19 Transmission	0.0000124
$\boldsymbol{\sigma^{\beta}}$	Rate of individual exposure	0.000011618
$\boldsymbol{\mu^{\beta}}$	Rate of natural deaths	0.00324588
$\boldsymbol{v_2^{\beta}}$	Individuals' rates of relapse	0.001466848
$\boldsymbol{\delta^{\beta}}$	COVID-19 mortality rate	0.00286
$\boldsymbol{\theta^{\beta}}$	Survival rate of individuals in quarantine	0.0766169
$\boldsymbol{\alpha^{\beta}}$	Rate of infectious individual detection	0.010939586
$\boldsymbol{v_1^{\beta}}$	The recovery rate of COVID-19 from infectious individuals	0.1109289
$\boldsymbol{v_3^{\beta}}$	The rate at which individuals who have been recovered return to the susceptible class	0.0022927

2.2. Construction of the NSFD Schemes for Fractional Epidemic Models

We build the NSFD schemes in this part in order to get numerical solutions for the fractional-order epidemic model (1). The fractional derivatives are approximated using the Grünwald–Letnikov method. The continuous fractional order epidemic model's fixed points in the solution are kept stable and positive by the suggested NSFD methods.

If one or more of the following criteria is met, a numerical scheme is referred to as an NSFD discretization:

1. A nonlocal approximation is utilized [31, 32].

2. Derivative discretization is not a conventional and nonnegative function [33]

$\psi(h) = h + O(h^2)$, called a denominator function is used [30]

$$\psi(h) = \frac{1-e^{-\lambda h}}{\lambda};\ \psi(h) = (1 - \cos \lambda h). \tag{3}$$

The elements on the right hand side of the fractional-order epidemic model (2) contain terms of the form S_n and S_{n+1} in order to generate a dynamically consistent NSFD scheme. This is how we construct the NSFD [29-34, 43-45] schemes using nonlocal approximation. This means that the solutions derived from the NSFD discrete scheme also need to satisfy the primary characteristics of the original system of differential equations or its solutions.

As previously established, the variables S, E, I, Q and R in the model indicate subpopulations and must always have positive values. Furthermore, $N(t)$ must equal the total of the subpopulations ($N(t) = 1$ for the scaled model). These characteristics must be maintained in the NSFD scheme in order for us to create the suitable scheme structure.

Let $N_0 \in \mathbb{N}$ and $[0, T]$ a finite interval. Let us denote by $h = \Delta t = \frac{T}{N_0}$ the time step size of the discretization,

$$0 = t_0 < t_1 < \cdots < t_{N_0} = T = N_0 h, \tag{4}$$

and let $t_n = nh$, with $n \in \{0, 1, 2, \ldots, N_0\}$. Then, S_n, E_n, I_n, Q_n and R_n correspond to the approximated values of $S(t_n)$, $E(t_n)$, $I(t_n)$, $Q(t_n)$ and $R(t_n)$, respectively.

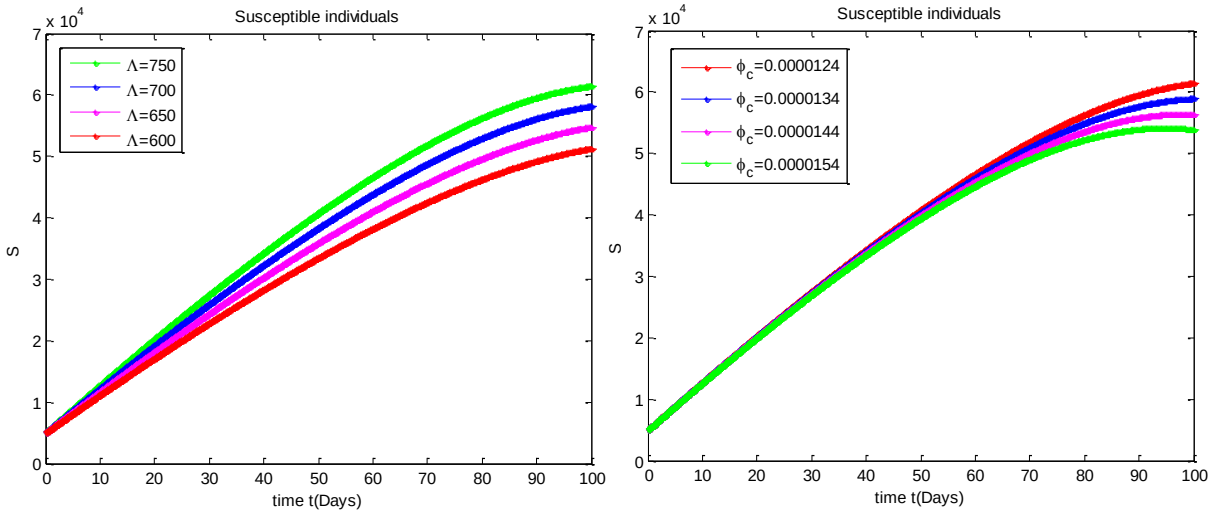


Figure 2. *Using the NSFD technique, the investigated model's susceptible $S(t)$ class behaves dynamically at various values of Λ and ϕ_c*

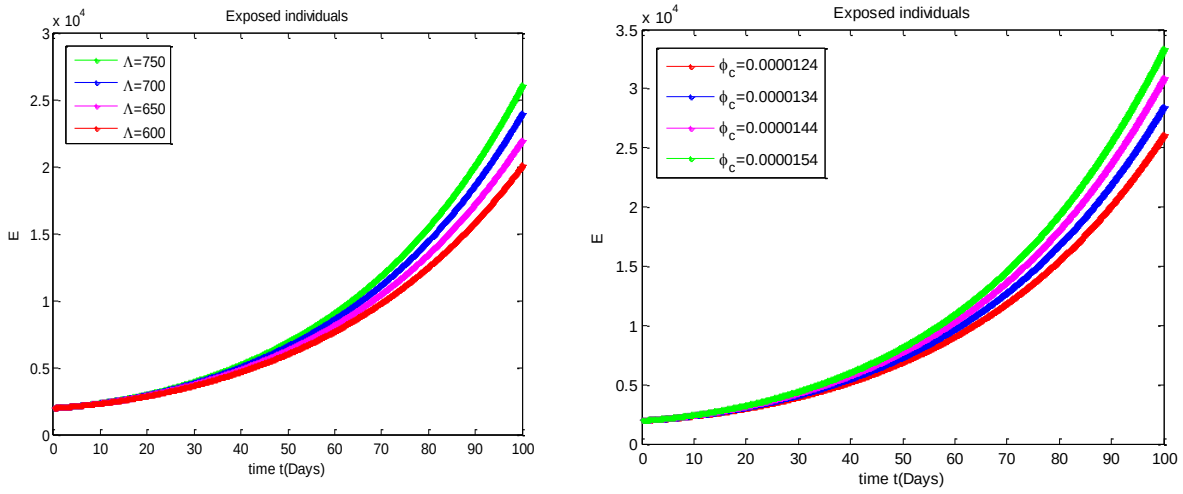


Figure 3. *The NSFD scheme was used to analyze the dynamic behavior of the model's Susceptible $E(t)$class at various Λ and ϕ_c values*

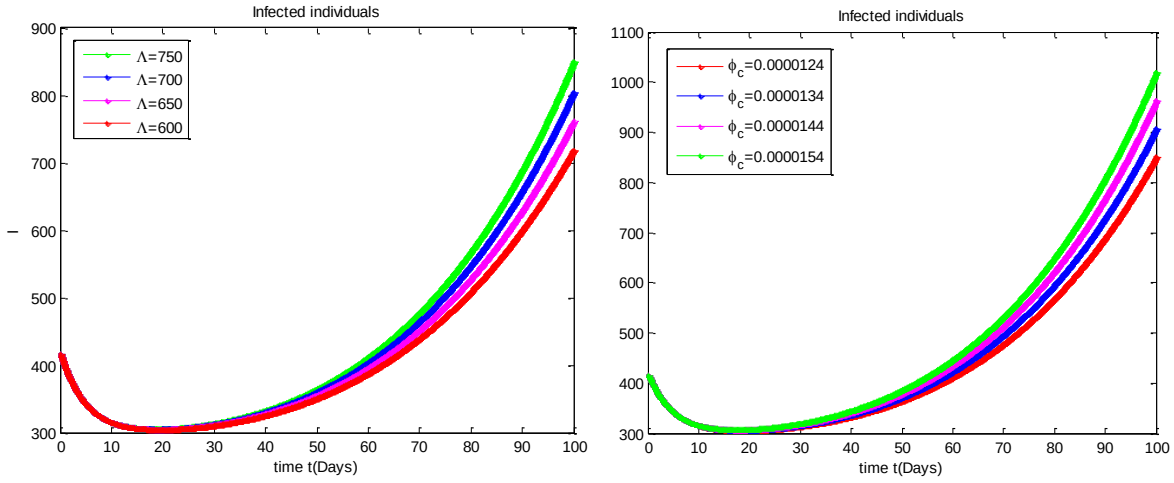


Figure 4. *Using the NSFD technique, the investigated model's susceptible $I(t)$ class behaves dynamically at various values of Λ and ϕ_c*

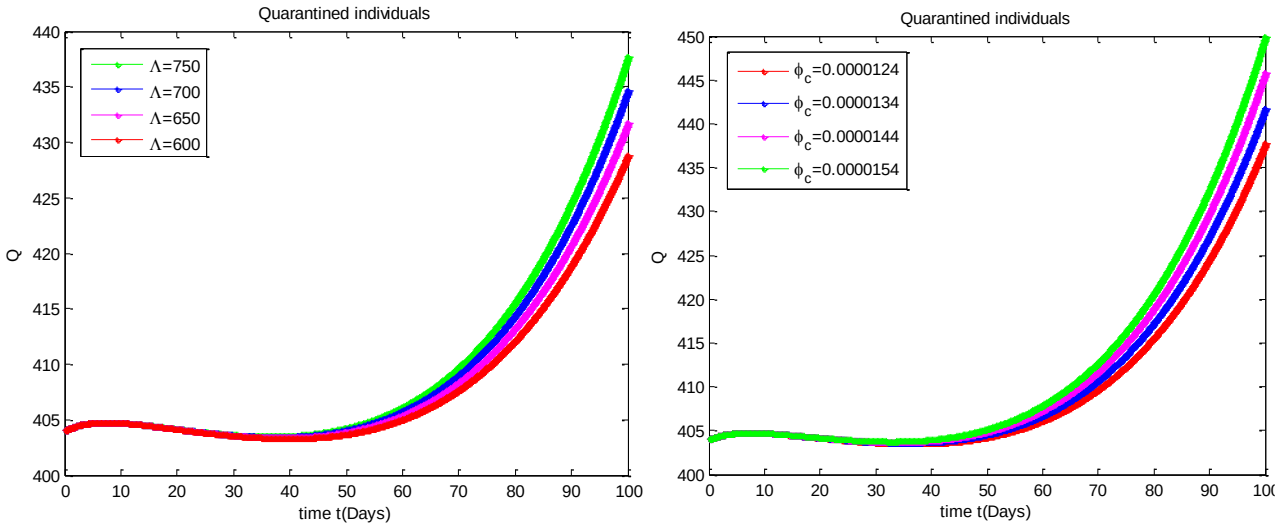


Figure 5. *Using the NSFD scheme, we can analyze the dynamic behavior of the model's Susceptible $Q(t)$ class at various Λ and ϕ_c values*

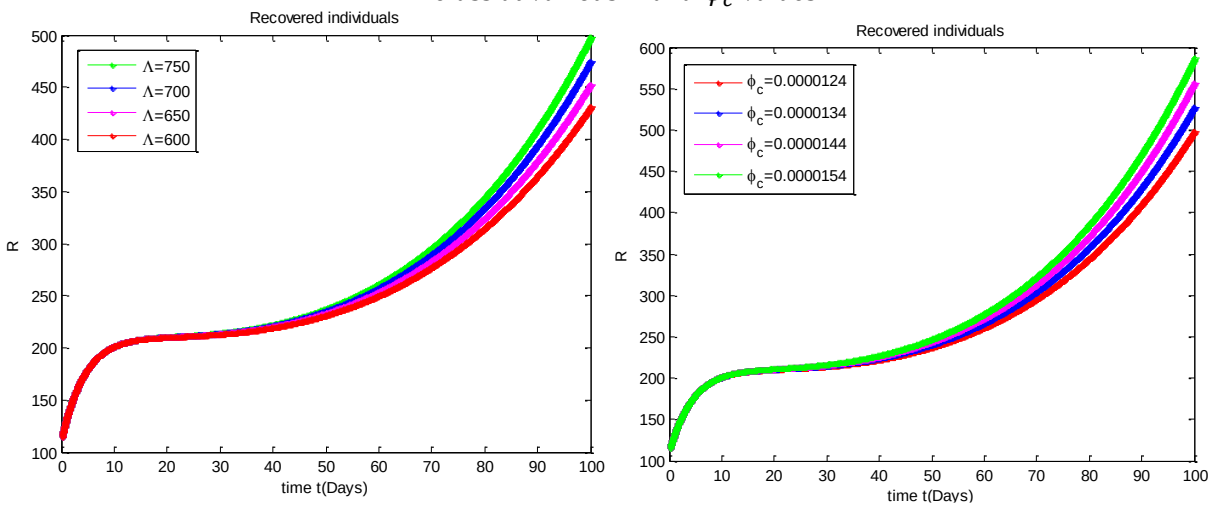


Figure 6. *The NSFD scheme is used to analyze the dynamic behavior of the considered model's susceptible $R(t)$ class for various values of Λ and ϕ_c*

Using the values from Table 2, we plotted the system (1). Using real data, we testify our system (1) for the real data with used Table 2 for the parameters values from 28 the February 2020 for coming 100 days. Using the NSFD scheme. In Figures 2–6, we investigate dynamic of transmission of Covid-19 model at different values of Λ and ϕ_c. Here for graphical interpretation, we take initial data as given in [1] as $(S_0, E_0, I_0, Q_0, R_0,)$ = (5000,2003,416,404,115) at 28 the February 2020. Now we simulate the numerical results of different compartments corresponding to different values of Λ and ϕ_c. The effect of in Figures 2-6, λ parameter on all compartments is the same, as the parameter decreases, the compartments also decrease.

Definition 4. The Caputo fractional derivative of order $\beta > 0$ of a function $\omega \in C_{-1}^{\mu}(\mu \geq 1)$ in the sense of Caputo is defined as

$$ {}_0^C D_x^{\beta} \omega(x) = \frac{1}{\Gamma(m-\beta)} \int_0^t (t-\vartheta)^{m-\beta-1} \omega^m(\vartheta) d\vartheta, \tag{5}$$

where $m-1 < \beta < m \in \mathbb{N}$. If $\beta = m \in \mathbb{N}$, then

$$D_t^{\beta}\omega(t) = \frac{d^m\omega(t)}{dt^m}.$$

Definition 5. The Laplace transform of function is defined as

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t)dt,$$

for all real numbers $t \geq 0$ if the integral exists.

Definition 6. [14] The Laplace transform of the Caputo fractional derivative $D_t^{\beta}\omega(t)$ is defined as

$$\mathcal{L}\left[D_t^{\beta}\omega(t)\right] = s^{\beta}\mathcal{L}[\omega(t)](s) - \sum_{k=0}^{m-1} s^{\beta-k-1}\omega^k(0^+), \tag{6}$$

where $\alpha > 0, m-1 < \alpha < m$ and $\mathcal{L}$ denotes the Laplace transform operator.

2.3. Numerical Analysis of Model (1) Using The Fractional Calculus Concept

In this section, solutions are obtained using the Grunwald-Letnikov fractional derivative algorithm for the numerical model of the fractional Covid-19 model. The significance of GL operators extends beyond historical and theoretical considerations; they also facilitate direct use in numerical computations by offering a discretization method. The Grunwald-Letnikov non-standard weighted average finite difference method was developed to simulate the fractional order system that was obtained. The basic integral and derivative formulas used in the Grunwald-Letnikov fractional derivative are given below.

2.3.1. Grünwald-Letnikov fractional integral and derivatives

Let $g(t)$ be given the continuous function. First, let the back difference derivative be given as follows.

$$\frac{d}{dt}g(t) \equiv g'(t) = \lim_{\hbar\to 0}\frac{g(t)-g(t-\hbar)}{\hbar}.$$

Using the general definition of the derivative, the second derivative of the function $g(t)$ is written in the form below

$$\begin{aligned}\frac{d^2}{dt^2}g(t) \equiv g''(t) &= \lim_{\hbar\to 0}\frac{g'(t)-g'(t-\hbar)}{\hbar}\\ &= \lim_{\hbar\to 0}\frac{1}{\hbar}\left\{\frac{g(t)-g(t-\hbar)}{\hbar} - \frac{g(t-\hbar)-g(t-2\hbar)}{\hbar}\right\}\\ &= \lim_{\hbar\to 0}\frac{g(t)-2g(t-\hbar)+g(t-2\hbar)}{\hbar^2}.\end{aligned}$$

With the help of the first, second and third order derivatives of the function $g(t)$ is

$$\frac{d^3}{dt^3}g(t) \equiv g'''(t) = \lim_{\hbar\to 0}\frac{g(t)-3g(t-\hbar)+3g(t-2\hbar)-g(t-3\hbar)}{\hbar^3}.$$

According to these rules, the general form of the nth order derivative of the $f(t)$ function for $\in N$, $j > n$ *i*s as follows.

$$\frac{d^n}{dt^n}g(t) \equiv g^{(n)}(t) = \lim_{\hbar\to 0}\frac{1}{\hbar^n}\sum_{j=0}^{n}(-1)^j\binom{n}{j}g(t-j\hbar). \tag{7}$$

For positive values of n, variable signs and binomial coefficients are defined as follows:

$$\binom{n}{j} = \frac{n(n-1)(n-2)\ldots(n-j+1)}{j!} = \frac{n!}{j!(n-j)!},$$

if n is negative,

$$\binom{-n}{j} = \frac{-n(-n-1)(-n-2)\ldots(-n-j+1)}{j!} = (-1)^j \begin{bmatrix} n \\ j \end{bmatrix},$$

it is possible. Here

$$\begin{bmatrix} n \\ j \end{bmatrix} = \frac{n(n+1)\ldots(n+j-1)}{j!},$$

is in the form. If $-n$ is substituted in the Equation (5), we get the following equation.

$$\frac{d^{-n}}{dt^{-n}} g(t) \equiv g^{(-n)}(t) = \lim_{h\to 0} \frac{1}{h^n} \sum_{j=0}^{n} \begin{bmatrix} n \\ j \end{bmatrix} g(t-jh).$$

Here n is a positive integer. Fractional derivative from general definitions of derivative and definition of derivative of order $\beta(\beta \in R)$ according to Equation (6),

$$D_t^{\beta} g(t) = \lim_{h\to 0} \frac{1}{h^{\beta}} \sum_{j=0}^{\infty} (-1)^j \binom{\beta}{j} g(t-jh),$$

can be written in the form. The relationship between the factorial and the Euler Gamma function can be used to calculate the binomial coefficients. From here we get $\binom{\beta}{0} = 1$ for,

$$\binom{\beta}{j} = \frac{\beta!}{j!\,(\beta-j)!} = \frac{\Gamma(\beta+1)}{\Gamma(j+1)\Gamma(\beta-j+1)}.$$

It is possible. Considering that $n = \frac{t-\beta}{h}$, where β is the real constant, the limit value expressed here can be written as follows

$${}_{\alpha}D_t^{\beta} g(t) = \lim_{h\to 0} \frac{1}{h^{\beta}} \sum_{j=0}^{\frac{t-\beta}{h}} (-1)^j \binom{\beta}{j} g(t-jh).$$

Let us now consider the fractional Covid-19 model

$$\begin{aligned}
&{}_0D_t^{\beta} S(t) = \Lambda^{\beta} - \phi_c^{\beta} I(t)S(t) - \mu^{\beta} S(t) + v_3^{\beta} R(t),\\
&{}_0D_t^{\beta} E(t) = \phi_c^{\beta} I(t)S(t) - \left(\sigma^{\beta} + \mu^{\beta}\right)E(t),\\
&{}_0D_t^{\beta} I(t) = \sigma^{\beta} \mathrm{E}(t) - \left(\alpha^{\beta} + v_2^{\beta} + \delta^{\beta} + \mu^{\beta}\right)\mathrm{I}(t) + v_1^{\beta} R(t),\\
&{}_0D_t^{\beta} Q(t) = \alpha^{\beta} I(t) - \left(\theta^{\beta} + \mu^{\beta}\right)Q(t),\\
&{}_0D_t^{\beta} R(t) = \theta^{\beta} Q(t) - \left(v_1^{\beta} + v_3^{\beta} + \mu^{\beta}\right)R(t) + v_2^{\beta} I(t).
\end{aligned}$$

where $0 < \beta \leq 1$,
$S(0) = S_0, E(0) = E_0, I(0) = I_0, Q(0) = Q_0, R(0) = R_0$ are initial conditions and $S(0) = 5000, E(0) = 2003, I(0) = 416, Q(0) = 404\,, R(0) = 115$ are real values [24].

The numerical solution of the fractional order Covid-19 model can be written in the form below.

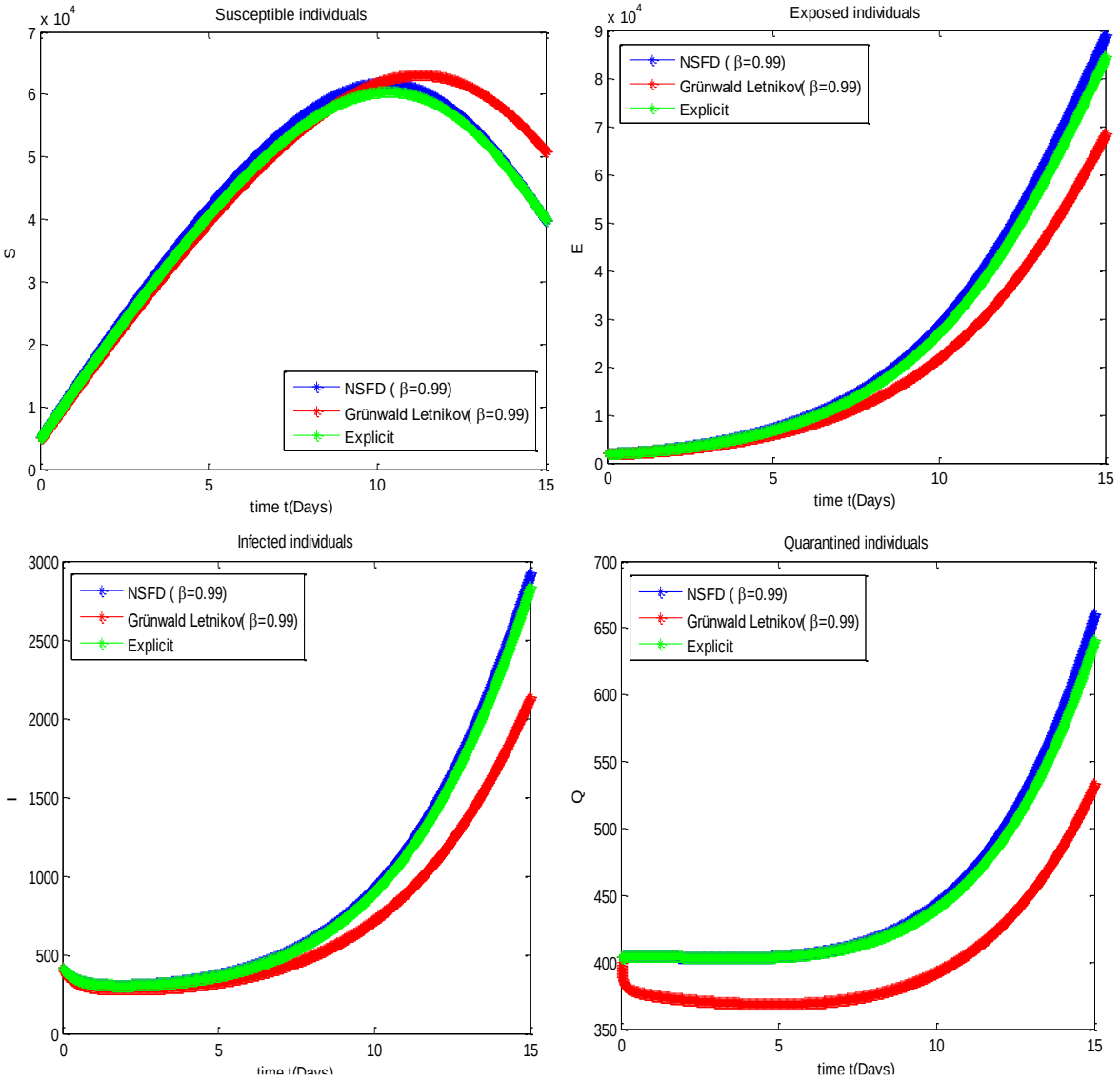
$$S(t_k) = \left(\Lambda^{\beta} - \phi_c^{\beta} I(t_{k-1})S(t_{k-1}) - \mu^{\beta} S(t_{k-1}) + v_3^{\beta} R(t_{k-1})\right)h^{\beta} - \sum_{j=v}^{k} c_j^{(\beta)} S(t_{k-j}),$$

$$E(t_k) = \left(\phi_c^{\beta} I(t_{k-1})S(t_k) - \left(\sigma^{\beta} + \mu^{\beta}\right)E(t_{k-1})\right)h^{\beta} - \sum_{j=v}^{k} c_j^{(\beta)} E(t_{k-j}),$$

$$I(t_k) = \left(\sigma^{\beta}\mathrm{E}(t_k) - \left(\alpha^{\beta} + v_2^{\beta} + \delta^{\beta} + \mu^{\beta}\right)\mathrm{I}(t_{k-1}) + v_1^{\beta} R\,(t_{k-1})\right) h^{\beta} - \sum_{j=v}^{k} c_j^{(\beta)} I(t_{k-j}),$$

$$Q(t_k) = \left(\alpha^{\beta} I(t_k) - \left(\theta^{\beta} + \mu^{\beta}\right) Q(t_{k-1})\right) h^{\beta} - \sum_{j=v}^{k} c_j^{(\beta)} Q(t_{k-j}),$$

$$R(t_k) = \theta^{\beta} Q(t_k) - \left(v_1^{\beta} + v_3^{\beta} + \mu^{\beta}\right) R(t_{k-1}) + v_2^{\beta} I(t_k) h^{\beta} - \sum_{j=v}^{k} c_j^{(\beta)} R(t_{k-j}).$$



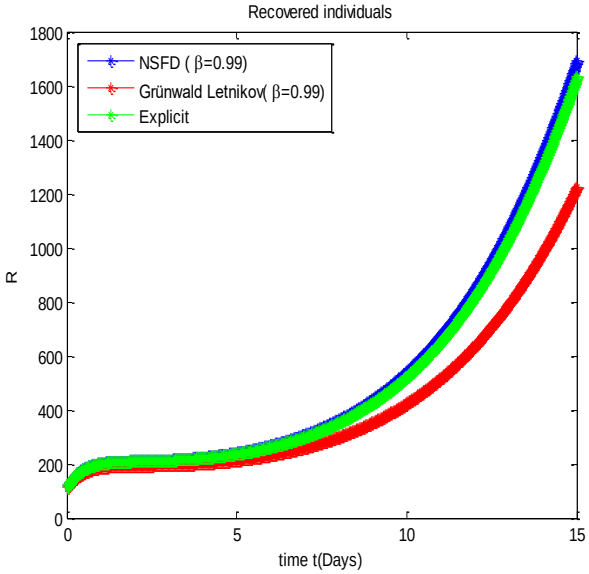


Figure 7. *Numerical solutions of the Covid-19 fractional model using explicit Euler* $(\beta = 1)$*, Grünwald-Letnikov and NSFD scheme with and without the denominator function*

$$\Psi(h) = (1 - \exp(-h)) \text{ for } \beta = 0.99 \text{ and } h = 0.001$$

Here T_{sim} simulation time is the initial conditions for $N = \left[\frac{T_{sim}}{h}\right], k = 1,2,3,\dots,N$ and $\big(S(0), E(0), I(0), Q(0), R(0)\big)$. $c_j^{(\beta_1)}$are binomial coefficients for each i.

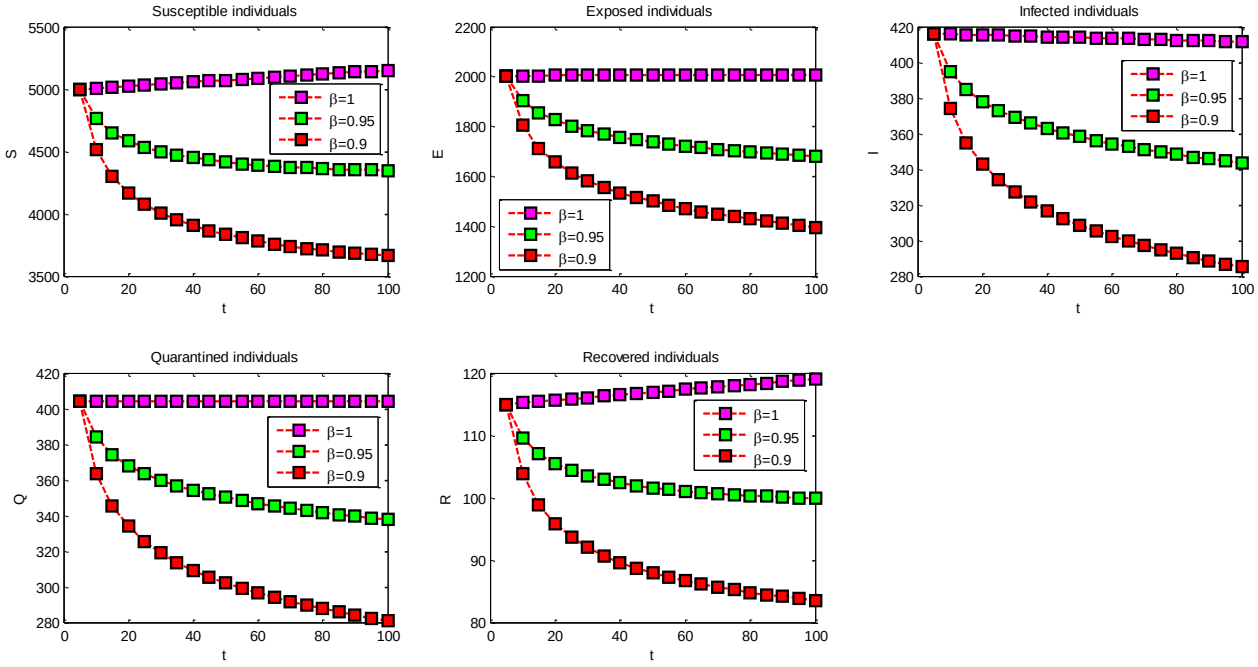


Figure 8. *Behavior of the fractional Covid19 model when stride length* $h = 5$ *and* $\beta = 1,0.95$ *and* 0.9 *in the* $[0\ 100]$ *time interval*

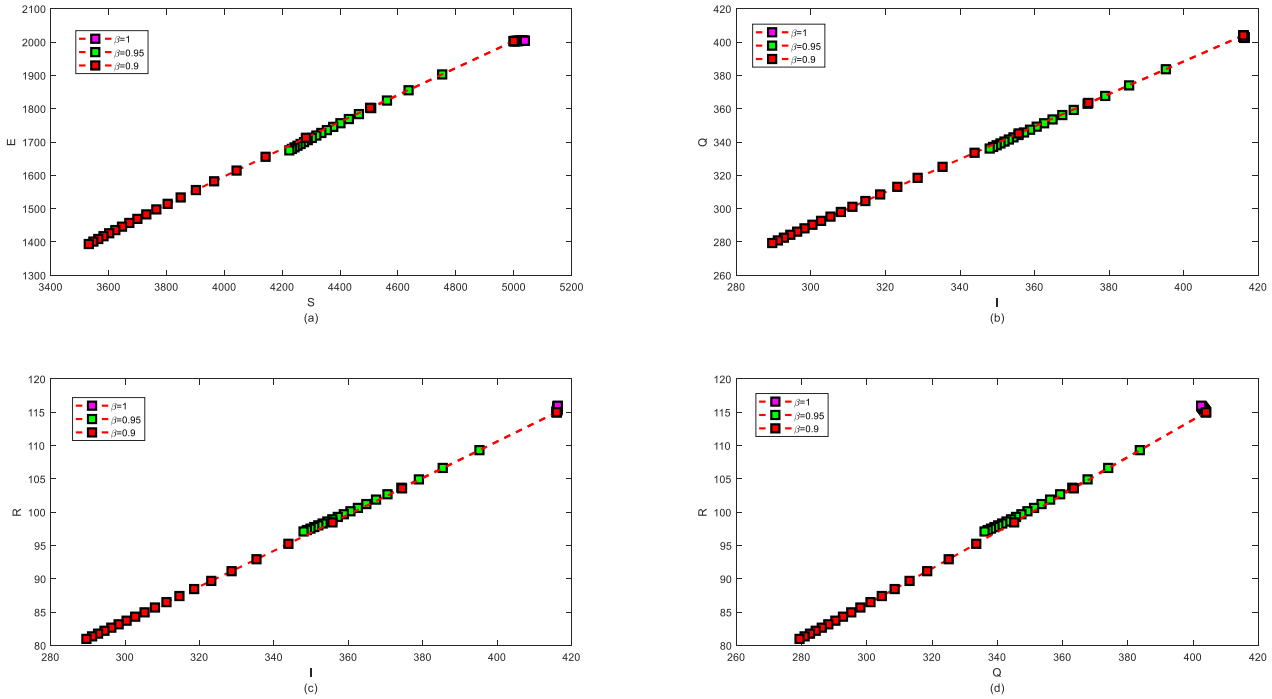


Figure 9. *Phase diagram for the numerical solution of the fractional Covid-19 model when the step length is* $h = 5$ *and* $\beta = 1{,}0.95$ *and* 0.9**.** $(a)S(t) - E(t);\ (b)I(t) - Q(t);$
$(c)I(t) - R(t);\ (d)Q(t) - R(t)$

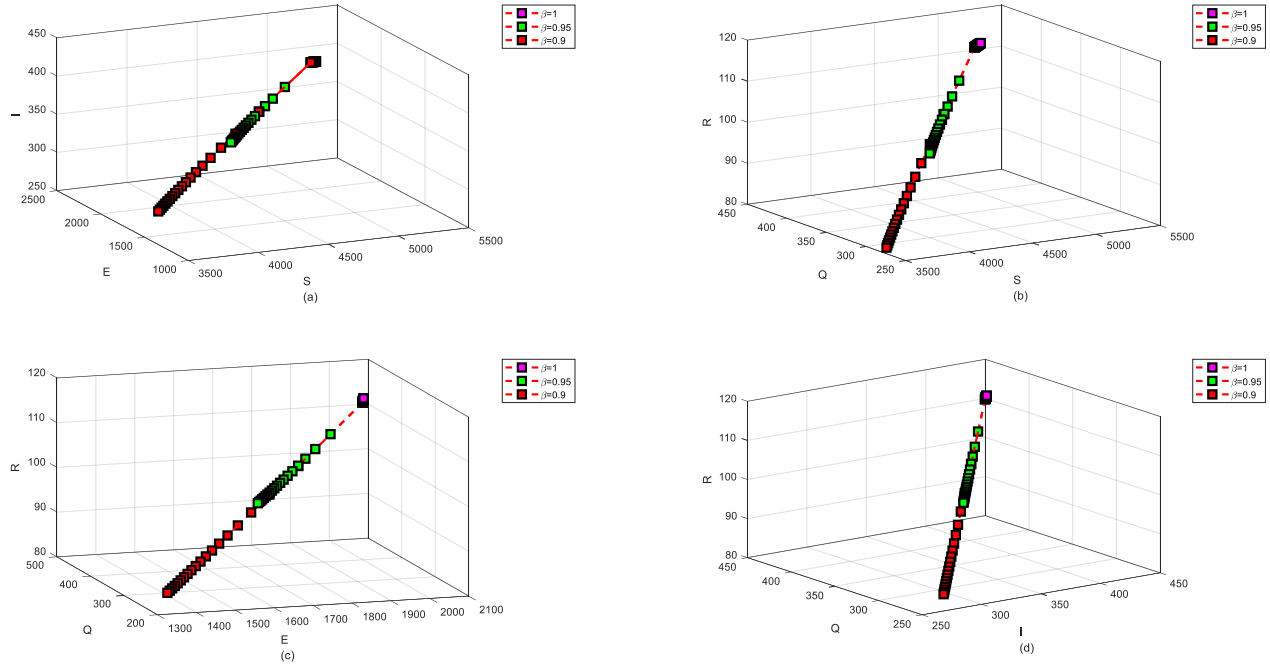


Figure 10. *Phase portrait of fractional Covid-19 model for numerical solution when stride length* $h = 5$ *and* $\beta = 1{,}0.95$ *and* 0.9

3. CONCLUSION

In this study, an NSFD scheme was created for the fractional Covid-19 model using the Grünwald-Letnikov decomposition. For different values of Λ and ϕ_c parameters, the dynamic behavior of the model was examined using the NSFD scheme. The proposed NSFD scheme was tested with several numerical simulations for different time intervals and fractional derivatives. Phase portrait and diagram of the model were drawn for different fractional derivatives. It has been seen that it is expcilit and inexpensive to compute to obtain positive numerical solutions of the fractional Covid-19 model with the NSFD scheme.

The convergence of the NSFD procedure is one of its advantages for larger step sizes. The NSFD method's efficacy for larger step sizes is presented in a tabular format. In the future, stability analyses can be conducted using a similar technique, and NSFD diagrams can be generated for linear and nonlinear models. Furthermore, the NSFD method can be employed to solve a variety of fractional COVID-19 models, as it can be applied to fractional order differential equations. Subsequently, we expanded the NSFD approach to encompass fractional order and analyzed the aforementioned model about fractional order dynamics. The analysis of Figures 7-10 indicates that the fractional order model is superior in elucidating conduction dynamics relative to the classical order model. The model can more effectively capture the complexities of the pandemic and provide more accurate and widely applicable insights by addressing these limitations, accounting for external factors such as mutations, environmental influences, and reinfections, and improving data collection and validation to increase the model's reliability.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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